The Study of Instructional Improvement

Instructional Log

Mathematics Glossary

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Mathematics Gateway Section

The log begins with general questions (Questions 1–4) about your mathematics instruction. These questions are what we refer to as the "gateway" section. Your answers to these questions will determine what other sections of the log you need to complete, making this section especially important.

Before you begin, find out who your target student is for the day. This student will be identified on your log each day, and you will report on the mathematics instructional experience of this target student.

Question 1: How much total time did the target student spend on mathematics today? Please include all mathematics instruction the target student received, including routine times such as morning or calendar math, even if the instruction took place in another room or by another teacher.

Mathematics includes work on topics such as number concepts, basic facts, computation, patterns, functions, algebra, geometry, measurement, probability, data analysis, percent, ratio, proportion, learning about money, telling time, and reading a calendar. It includes work with whole numbers, fractions, decimals, and negative numbers. Mark the total amount of time the student spent in instruction when mathematics was the focus. When mathematics was the focus during more than one period of instruction, please add together and report the cumulative amount of time the student spent in mathematics instruction. This includes time the student spent in mathematics instruction with other teachers such as Title 1 teachers, special education teachers, or tutors. It also includes computer lab time that was spent on mathematics instruction and practice. Mathematics instruction might include an individual student or a small group of students working for a sustained period of time on mathematics while other students work on other subject matter.

Please do not use this log to report on times when mathematics was done by the student, but was not a focus of the lesson. For example, **do not use the log to report on science lessons where students were asked to do some computation, but the mathematical concepts or operations were not a focus**. That is, if mathematics was needed to complete an assignment, but the focus of the instruction was on the science or social studies content rather than on how to solve mathematical problems, do not include this time. However, if the mathematics was a focus of the lesson (e.g., you explained how to use a formula or how to compute the area of a region), include the time in Question 1 and report on the areas taught.

Include times the target student was present, participated in instruction, and/or was expected to do work with a focus on mathematics. Do not include non-instructional time such as transitions, late starts, and major interruptions (e.g., fire drills, long announcements, bathroom breaks). Please add together the individual blocks of time and record the total time the target student was involved in mathematics instruction on the log form.

How to report on just one student: Our questions ask about the target student, not about you or your whole class. For our purposes, however, if you asked the whole class to do something, please consider the target student "engaged" in that task – regardless of whether you heard or saw the target student's thoughts or work, and whether or not the target student appeared to be working or not. For instance, if you asked the whole class to reason about a new idea and several non-target students volunteered their thoughts, you may check "Perform tasks requiring ideas or methods not already introduced to the student" for the target student as well. What is important is that the target student was asked as part of the class to think about this idea, and had the opportunity to do so, even if he or she did not volunteer an answer.

Note: If the target student did not have mathematics today, it is still important that you complete Questions 1 and 3. After that, you are finished with mathematics logging for the day.

Question 2: Of the mathematics time recorded in Question 1, how much time were you either the teacher or an observer of the teaching?

In this log we would like you to report in more detail on the instruction that you provided to the target student as well as on instruction of which you have first-hand knowledge. This includes your own personal teaching as well as instruction that you plan and supervise. For example, you might have provided instruction through assigning practice exercises, by team-teaching with another teacher, or by planning an instructional activity and directly supervising an aide (or student teacher) as the aide delivered the instruction. Please record the actual cumulative amount of time the target student was involved in mathematics instruction under your direct supervision.

Question 3: Please mark the reason(s) why you recorded 0 minutes in Question 1 or 2, and then stop here.

If you answered "0" to Questions 1 or 2, please explain the reason(s). The reasons listed in this question are those reasons most commonly given during our pretest work. If the reason that the student did not have instruction with you is not listed, please mark "Other" and briefly indicate the reason.

Note: The response "Participating in testing or test preparation" refers to district-wide or school-wide standardized testing programs. **Please do not include mathematics assessments that you administered for instructional purposes. If you conducted your own mathematics assessment, please report the topic assessed in Question 4**. For instance, if you administered an assessment on multi-digit multiplication for your own use, you would not record that in Question 3. Instead, you would record it in Question 4 (Operations), then move to the Operations section of the log.

If you marked "0" minutes in Questions 1 or 2, and then completed Question 3, you are finished with today's log.

Question 4: To what extent were the following topics a focus of your work with the target student in mathematics today?

This question asks which topics were emphasized in the target student's mathematics instruction today. Please go through the entire list and check all the topics encountered by the target student. However, please remember to consider what actually happened during the day's instruction, not what you had planned. Your topic selections determine which of the remaining sections of the mathematics instructional log you will be asked to complete.

As you consider what you taught, please record how much focus was placed on each topic. Topics can be taught in different combinations and with different degrees of emphasis. On some days you might teach multiple topics, while on other days you might teach just one.

A major focus

Use this category if the topic was a main emphasis or a primary teaching objective in the mathematics instruction experienced by the target student. You might have more than one area of major focus on a given day. The major focus for the day might have been new material that you were introducing, or it might have been review or practice to which you devoted a large percentage of the mathematics instruction.

A minor focus

Use this category if the topic was a focus of instruction for the target student, but was not a primary emphasis or your main teaching objective. Instruction or practice in this topic area might have been a smaller feature of the lesson or supportive of your teaching in the area(s) of major focus. Your minor focus might have been directly related to your major focus or it might have been another area. For example, the major focus of your instruction might have been multi-digit addition, but you might have asked the target student several questions about place value while working on that topic. Or you might have focused primarily on patterns, but spent some time earlier in the day reviewing basic number facts. Typically, less of your instructional time would be spent on areas that you consider a minor focus. You might have more than one area of minor focus on a given day.

Touched on briefly

Use this category if the target student only briefly engaged with a topic. For example, the student might have briefly reviewed basic fact combinations or named geometric shapes on a bulletin board.

Not taught today

You should mark a response in every line of Question 4, one for each topic. If you did not choose A major focus, A minor focus, or Touched on briefly, please indicate that this area was not taught today.

How to decide whether or not to record a mathematics topic: Many different mathematics topics may arise in the course of a single lesson. We are not asking you to record every topic that comes up. Please use the following principles to decide whether or not to record a topic: If you were focusing on one topic (e.g., patterns) and another topic (e.g., multiplying to check the pattern) was implicit in the first topic, please do not also record the secondary topic. This rule applies only to situations in which the second task was merely to help the instruction and was not a focus of instruction or intended practice – for instance, finding single-digit combinations in the course of doing multi-digit addition, or reducing fractions in the course of instruction on multiplying fractions. However, if the second task was a focus of the day's instruction or was intended student practice, please mark both.

4a. Number concepts (whole number, decimal, or fraction)

Number concepts refers to all non-computational work on whole numbers, decimals, or fractions. This includes writing, reading, or naming numbers; counting; comparing or ordering quantities; understanding place value; relationships between fractions and decimals; and estimating. For whole numbers only, it also includes properties of numbers (such as odd and even, prime and composite, square numbers), and factors, multiples, or divisibility. For fractions, it also includes work on the meaning of a fraction, on equivalent fractions, and on simplifying fractions. **Please do not mark work on computation, basic facts, or patterns here unless that work was accompanied by a significant piece of work on a number concept topic as well.**

4b. Operations (whole number, decimal, or fraction)

Operations refers to work on addition, subtraction, multiplication, and division. Include any work on meanings of these operations, understanding and developing competency with basic facts, multi-digit computation with whole numbers, and any computation with decimals or fractions. **Please do not record operations with negative numbers here – instead please mark d7 below**.

4c. Patterns, functions, or algebra

Patterns, functions, or algebra includes work on organizing objects by size, number, or other properties into groups, categories, or lists; different types of patterns; generalizing patterns; using symbols to express unknown and variable quantities; and understanding and using formulas. A function is a relation that expresses how one quantity or variable changes with respect to another. For example, the area of a square can be expressed as $A = s^2$ where s is the length of one side. We say that the area of a square is a function of its side length because the function, expressed as the formula $A = s^2$, shows how the area (A) varies in relation to variation in the length of the side (s).

4d. Other mathematical content

Mathematics instruction that does not fit into one of the above focal topics should be marked here. Here are definitions and/or examples to help clarify these topics:

1. **Learning about money, telling time, or reading a calendar**
   Include in this category only instruction about features of money, time, or the calendar **– not instruction that merely uses these to help students practice facts or procedures.** For instance, include instruction about the value of money, but do not include multi-digit computation problems that use money as a context – please record the latter above in "operations." By "reading a calendar" we mean instruction that helps students understand the ideas of days, weeks, months, and the construction of the calendar, but not work which uses the calendar to help students explore or practice other mathematical concepts or procedures.

2. **Representing or interpreting data**
   Include in this category work on creating or using tallies, tables, graphs or charts to represent data; making inferences or drawing conclusions from data; and lessons on mean, median, or mode.

3. **Geometry**
   Include in this category instruction about geometry – area and perimeter, shapes, angles, points and lines, and spatial reasoning. Also include instruction that covers geometric concepts, such as parallel, perpendicular, or congruent. If students are studying geometric designs (e.g., tessellations) please also report that here.

4. **Measurement**
   Include in this category instruction about length, weight, volume or capacity, units of measurement, and systems of measurement (e.g., metric, English).

5. **Probability**
   Include work on the concept of probability, methods of estimating or calculating the likelihood of different outcomes, or problems involving combinations and permutations.

6. **Percent, ratio, or proportion**
   Include here any work done with concepts of percent, ratio, or proportion, as well as computation that involves percent, ratio, or proportion, or applications of these concepts.

7. **Negative numbers**
   Include any work on the meaning of negative numbers, or on computation or applications involving negative numbers.

8. **Other**
   Please mark here for any topic that is not listed above, and write the topic on the line provided.

If you were working on problem solving: Please mark those activities under the topic area(s) corresponding to the problem(s) (e.g., geometry, operations, patterns) and then, if necessary, continue on to the focal topic area. You will be able to provide more detail about your problem solving activities when the content of those activities takes you into one of the focal topics.

If you administered an assessment: Please mark that activity under the topic area(s) corresponding to those on the assessment. For instance, if you administered an assessment on multi-digit addition and subtraction, and assessed students' knowledge through computational exercises like 24 + 38 = 62 and several word problems, you would mark Operations in Question 4, then proceed to the Operations section of the log. On days you administered assessments on a focal topic, please fill out the focal topic sections of the log based on the content of the assessment – the sub-topic being assessed, any materials used, and the types of student tasks involved (e.g., performing tasks using methods known to the student, or assessing a problem and determining a method to use). When we say assessment, we mean to include any type of quiz or test you use with your students for your own purposes (e.g., a two-minute basic fact quiz).

Mathematics Focal Topics

Questions A1, B1 and C1: What were you using in your work?

These items ask about what you worked with in today's instruction: whole numbers, decimals, or fractions.

The term **"whole numbers"** refers to positive integers including zero (0, 1, 2, …).

The term **"decimals"** refers to numbers represented in decimal form, with or without digits to the left of the decimal point (e.g., 3.50 or .45).

The term **"fractions"** refers to numbers represented in the form $\frac{a}{b}$, where a and b are positive integers, and b is not 0. These include fractions less than 1, such as $\frac{3}{5}$, and fractions greater than 1, written either as $\frac{18}{7}$ or $2\frac{4}{7}$.

Questions A2, B2, and C2: What did the target student work on today?

These items ask about specific activities that you might have done with the target student in the course of working on a focal topic. Not all activities will apply to your grade level. Please use the following guidelines in determining a topic's emphasis:

A focus of instruction

Use this category to represent topics if they received sustained attention in today's instruction. By sustained, we mean more than a brief comment or several short-answer questions about a topic. Instead, the target student should have had a significant opportunity to learn about the topic described. For instance, the target student might have reviewed basic facts during one segment of instruction, worked on addition and subtraction word problems for another segment of instruction, and collected data on his classmates' favorite flavors of ice cream in another segment. Each of these topics and activities could be marked "a focus of instruction." There is no specific time criterion for whether a topic is a focus of instruction. Please use your judgment, taking into account the time that the student spent on the topic and the importance of the topic to the day's work.

Touched on briefly

Use this category to represent topics in which the target student was engaged for a moment or two. Examples include a short review of reducing fractions in the midst of working on addition of fractions, or a few questions to practice basic facts in a lesson on multiplying two-digit numbers. It can also include topics that come up when a student's question leads you to spend a short amount of time on a topic.

Section A: Number Concepts

A2a. Writing, reading or recognizing whole numbers, decimals, or fractions

Use this category to represent work on understanding written representations of whole numbers, decimals, or fractions. For instance, the target student might have learned to read 104, to say 1.23 as one and twenty-three hundredths, or to write three and two-thirds as $3\frac{2}{3}$. This category can also include writing, reading, or recognizing numbers written with exponents, such as learning that 2 x 2 x 2 can be written as 2^3.

A2b. Counting

Use this category for any work on counting with whole numbers, decimals, or fractions and/or telling how many items are in a set. This might include rote counting, skip counting (e.g., 5, 10, 15, 20…), matching sets, counting with one-to-one correspondence, having students identify the number of items in a set (e.g., five balloons or three and a half cookies), or another activity where counting is the primary goal. **Please do <u>not</u> mark here if the counting occurred merely to help with another topic. For example, if a student counted to help learn basic addition facts, please mark that under basic facts – B2b or B2c.**

A2c. Comparing or ordering two or more quantities

Use this category to represent work on comparing or ordering two or more quantities, such as using concepts of equal to, greater than, or less than to compare quantities, or ordering a group of whole numbers, decimals, or fractions from least to greatest.

A2d. Properties of whole numbers (e.g., even and odd, prime, square)

Use this category to represent work on the characteristics of whole numbers. For instance, classroom work might have focused on the differences between odd and even, prime and composite numbers, or other means of categorizing numbers. **However, if you are working on properties of an operation (e.g., the commutative property of addition), please mark that under the "Operations" section.**

A2e. Factors, multiples, or divisibility with whole numbers

Use this category to represent work on factors, multiples, or divisibility with whole numbers (not with decimals or fractions). **Please do not mark skip counting here – instead please mark it in counting. If instruction or practice focused on the use of multiples for working on basic facts, please mark it in basic facts unless you explicitly focused on the idea of multiples during instruction. Also do not mark work on division facts (e.g., 24 ÷ 6) or computation (e.g., 248 ÷ 16) here unless you explicitly focused on concepts of divisibility (e.g., finding all the divisors of 24).**

A2f. Composing or decomposing (grouping) whole numbers or decimals into tenths, ones, tens, hundreds, etc.

Use this category to represent work on grouping, trading, composing, or decomposing whole numbers or decimals. Please include here work on writing numbers in expanded notation (e.g., 365 = 300 + 60 + 5). In another example of such work, the target student might have had 35 single cubes, then traded 30 of them for three "tens" sticks. Or, the target student might have worked to rename a number like 4.65 using place value concepts (e.g., 4.65 can be 465 hundredths or 46.5 tenths).

A2g. Identifying the values of the places in whole numbers or decimals

Use this category to represent work on the value of the places in written numbers. For instance, you might have worked on the idea that the 2 in 27 represents 20, or you might have worked on identifying the value of the 3 in the number 14.03 as three-hundredths. Also include here work on the relative value of places (e.g., the hundreds place is ten times the tens place).

A2h. The meaning of fractions

Use this category to represent work on the meaning of fractions. This might include interpretations of fractions as part of a whole, as part of a set, as points on a number line, or as division. Work on what is represented by the numerator or the denominator should also be recorded here. For example, the target student might have learned to interpret $\frac{3}{4}$ or $3\frac{1}{4}$. Or the target student might have learned that the denominator of a fraction indicates the number of equal parts a region (or object) is divided into and that the numerator indicates how many of the parts to include.

A2i. Understanding equivalent fractions or working on reducing fractions

Use this category to represent work in which you or the target student explained or showed how two or more fractions are equal, or learned or practiced the procedure for finding equivalent fractions or the procedure for reducing fractions. Also use this category when working on converting back and forth between writing fractions that are greater than 1 in different forms (e.g., writing $3\frac{1}{4}$ as $\frac{13}{4}$).

A2j. Relationships between decimals and fractions

Use this category to represent work that illustrates linkages between decimals and fractions. For example, the target student might have been asked to rewrite $\frac{1}{4}$ as 0.25 or to write 3.125 as $3\frac{1}{8}$.

A2k. Estimating the size of quantities or rounding off numbers

Use this category to represent work on estimating the size of quantities, comparing numbers to benchmarks, or rounding off numbers. For instance, the target student might have determined whether 67 is closer to 50 or 100, or whether 0.309 is closer to .25 or to .5, or whether $\frac{4}{9}$ is more or less than one-half, or he or she might have rounded 2.34 to the nearest tenth.

Section B: Operations

B2a. The meaning or properties of an operation

Use this category to record work on what it means to add, subtract, multiply, or divide, or work on the properties of these operations. For instance, you might have shown how subtraction can be represented as taking away one quantity from another, as comparing two quantities, or both. Or you might have explained 15 ÷ 3 as the process of partitioning 15 into three groups, or making groups of three out of 15. Also include here explicit work on the properties of operations (e.g., commutativity or associativity of addition and multiplication, or the principle that a number is not changed when zero is added to it or when it is multiplied by one), or the relationships of operations to each other (e.g., that subtraction is the inverse of addition). **Please do <u>not</u> use this category to represent times you or the target student worked <u>only</u> on developing strategies to find answers, or practicing facts or procedures (record these below in B2b or B2c).**

Basic facts (whole numbers only):

Basic facts are operation combinations that students typically attempt to memorize or learn to do quickly and mentally. For addition, basic facts include the sum of any two numbers from 0 to 10. For subtraction, basic facts include all the problems that correspond to the addition facts (e.g., 18 - 9 or 7 – 4, **but <u>not</u>** 18 – 4). For multiplication, basic facts include the product of any two numbers from 0 to 12. For division, basic facts include all the problems that correspond to the multiplication facts (e.g., 24 ÷ 6 or 63 ÷ 9, **but <u>not</u>** 72 ÷ 4).

B2b. Methods or strategies for finding answers to basic facts

Use this category to represent work on developing ways to remember or compute basic facts**, but <u>not</u> if you <u>only</u> practiced facts for speed or accuracy.** For example, the target student might have learned about a "fact family" like the + 9s or x 5s, or worked to recognize 17 as one more than the double 8 + 8 – "doubles plus one." The student might also have worked with manipulatives, number lines, or an abacus to figure out facts.

B2c. Practicing basic facts for speed or accuracy

Use this category to represent work on becoming more accurate or fast with basic facts. The target student might have used flashcards, worksheets, textbooks, games, or other means of practicing basic facts.

Computation with multi-digit whole numbers, decimals, or fractions:

Computation with multi-digit whole numbers comprises whole number addition, subtraction, multiplication, or division beyond basic facts (e.g., 8 + 14, 23 – 4, 12 x 14, 81 ÷ 3). Computation with decimals or fractions includes any addition, subtraction, multiplication, or division with any type of decimal or fraction (e.g., .2 + .2 or $\frac{1}{2} + \frac{1}{3}$).

B2d. Why a conventional computation procedure works

Use this category to record work on exploring why a conventional computation procedure works. For instance, when teaching the problem 53 – 29, you or the target student might have used blocks to turn 53 into 40 + 13, explaining that 40 + 13 is another way of

representing 53, which makes it possible to subtract the nine in the units column. Or when adding decimals, you might have shown students that lining up the decimal point allows you to combine tenths with tenths, hundredths with hundredths, etc. **If you simply explained the steps or walked through a procedure and did not explain why they work, please mark that in B2e or B2g.**

What we mean by conventional computational procedures: The following are examples of conventional multi-digit whole number procedures.

$$\begin{array}{r} {\scriptstyle 1} \\ 53 \\ +19 \\ \hline 72 \end{array} \qquad \begin{array}{r} {\scriptstyle 4\ 13} \\ 5\!\!\not 3 \\ -19 \\ \hline 34 \end{array} \qquad \begin{array}{r} {\scriptstyle 2} \\ 53 \\ \times\ 9 \\ \hline 477 \end{array} \qquad \begin{array}{r} 58\ \mathbf{R}8 \\ 9\overline{)530} \\ \underline{45} \\ 80 \\ \underline{72} \\ 8 \end{array}$$

Please also include close variants of these procedures (e.g., using different notation to keep track of borrowing or trading or different notation for bringing down numbers in long division).

Conventional decimal procedures look very much like their whole number counterparts. For addition and subtraction, students line up the decimal points and then proceed as above. For multiplication, students proceed as above and then add in the decimal point after counting the number of decimal digits in the two operands. For division, the students eliminate the decimal point from the divisor by moving it to the right and then move the decimal point of the dividend the same number of places.

Here are some examples of conventional fraction procedures:

$$\frac{2}{5}+\frac{5}{10}=\frac{4}{10}+\frac{5}{10}=\frac{9}{10} \qquad 2\frac{1}{2}\times 2=\frac{5}{2}\times\frac{2}{1}=\frac{10}{2}=\frac{5}{1}=5$$

$$3\frac{1}{4}-1\frac{3}{4}=2\frac{5}{4}-1\frac{3}{4}=1\frac{2}{4}=1\frac{1}{2} \qquad \frac{1}{2}\div\frac{1}{8}=\frac{1}{2}\times\frac{8}{1}=\frac{8}{2}=\frac{4}{1}=4$$

B2e. How to carry out the steps of a conventional computation procedure

Use this category to represent work on following steps to complete computation problems. The target student should have worked to master the steps in the procedure, **not yet striving for speed or accuracy.** For instance, you might introduce multiplication with decimals, and the student might work that day to follow the steps of this procedure correctly**. If teaching alternative (or non-conventional) procedures, record in B2g.** Please see the box above for what we mean by conventional procedures.

B2f. Practicing computation procedures for speed, accuracy, or ease of use

Use this category to represent work on helping the target student increase the speed, accuracy, or ease of use in following procedures for computation. The target student might have used flashcards, worksheets, textbooks, games, mental math, or other means of practicing computation procedures.

B2g. Developing transitional, alternative, or non-conventional methods for doing computation

Use this category to represent work on learning, using, or inventing non-conventional methods for computing with whole numbers, decimals or fractions. These include methods that differ from those described above as conventional procedures. Work with non-conventional methods may be informal, such as adding 53 + 19 by "rounding and compensating" (i.e., 53 + 20 = 73; 73 – 1 = 72) or mentally adding the tens before the ones. Other non-conventional methods may help the target student "see" the steps in an operation more clearly, as in these non-conventional methods for multi-digit multiplication and division:

$$\begin{array}{r} 231 \\ \times\ 4 \\ \hline 4 \quad (1\times 4) \\ 120 \quad (30\times 4) \\ \underline{800} \quad (200\times 4) \\ 924 \end{array} \qquad \begin{array}{r} 17\overline{)294} \\ \underline{170} \quad (17\times \mathbf{10}) \\ 124 \\ \underline{85} \quad (17\times \mathbf{5}) \\ 39 \\ \underline{34} \quad (17\times \mathbf{2}) \\ 5 \end{array}$$

$$10 + 5 + 2 = 17\ \mathbf{R}5$$

With decimal multiplication, the target student might use a non-conventional multiplication procedure and/or might use a non-conventional approach to place the decimal point in the answer. For example, the target student might estimate what would make sense in terms of the size of the answer instead of counting the number of decimal digits.

With fractions, the target student might do $2\frac{1}{2}$ x 2 by doing 2 x 2 + 2 x $\frac{1}{2}$, for example, or the student might do 3 divided by $\frac{1}{2}$ either mentally or by using pictures and asking how many $\frac{1}{2}$'s go into 3 wholes.

B2h. Applying basic facts or computation to solve word problems or puzzles

Use this category to record work on using basic fact or computation skills to solve word problems or numerical reasoning problems. By word problems, we mean problems written to represent real-life or imagined situations in which mathematics might arise. For example, the student might have solved the problem "How many eggs are in three twelve-egg cartons?" or the problem "If Jordan

drove 26 miles and has another 14 miles to go, what is the total distance of his trip?" By puzzles, we mean tasks in which students use their knowledge of basic facts or computation to reason about logically or deduce an answer to a numerical or contextualized problem. For example, the student might have worked on a "magic square," where all the rows, columns, and diagonals total the same sum. Or the student might have tried to figure out how many ways there are to exchange nickels and dimes for a quarter. Generally, these are problems that involve a number of numerical and computational clues to figure out a specific answer. **However, if the object of the work was the practice of basic facts or computational procedures to improve speed, accuracy, or ease of use, please mark B2c or B2f instead.**

B2i. Estimating the answer to a computation problem

Use this category to represent work on estimating answer(s) to computation problem(s). For example, in doing 23 + 39, the target student might have rounded off to 20 + 40 = 60 to get an approximate answer. This might be done when an estimate is sufficient, or it might be done to check the reasonableness of an obtained answer. It also might be done in applications, for example, "John has $3.53 and Maria has $2.40. About how much money do they have altogether?" If you asked the target student to determine an approximate or reasonable answer to a calculation, please mark that here.

Section C: Patterns, Functions, or Algebra

C2a. Organizing objects by size, number, or other properties

Use this category to represent work on sorting, classifying, or ordering objects in any way. For example, you might have asked the target student to group a set of buttons by color, by size, by shape, or by any other characteristic.

Types of patterns

C2b. Creating, continuing or explaining *repeating* patterns (e.g., 2,1,2,1,2,1… or □ ,△ ,○ ,□ ,△ ,○ …)

A repeating pattern is a simple pattern that repeats indefinitely. In other words, a set of terms is repeated over and over again. These patterns are sometimes described using letters (e.g., ABAB or ABCABC). The target student might have been asked to replicate, continue or explain repeating patterns in one of many ways, including clapping, orally, or with pictures or objects. In addition, students might have been asked to create their own repeating patterns. Please log here if the goal was to develop students' ideas about what a pattern is.

C2c. Constructing *sequences*, explaining their patterns, or predicting subsequent terms (e.g., 3, 7, 11, 15…)

A sequence is an ordered list of numbers, shapes, or other mathematical objects. Here we mean patterned sequences of continued growth, or change, as opposed to repeating pattern sequences, which would be recorded above in C2b. You might have asked your class to build, replicate, or continue a sequence of numbers or shapes by stating or writing the next few terms. Or students might have been asked to explain the rule for how a sequence is growing, for example, that numbers are going up by 4 each time or that there are two more triangles added each time. Please include geometric growth patterns as well as numeric growth patterns. Numeric sequences are <u>not</u> necessarily always increasing uniformly or in a linear fashion. For example, the target student might have identified the "growth pattern" in (0, 4, 3, 7, 6, 10…) as "plus four, then minus one," repeated over and over.

C2d. Finding and explaining *other patterns* (e.g., patterns in a representation like the hundreds chart or patterns in a word problem)

Use this category for patterns that are connected to a mathematical representation or problem. Here we mean to include all work on finding or explaining patterns, **except for repeating patterns or sequences, which would be recorded above in C2b or C2c.** For example, in problems with multiple solutions, the target student might have learned to make tables to organize those solutions and then look for patterns within the table. Or the student might have found patterns in an addition or multiplication table.

C2e. The use of a symbol to stand for an unknown number (e.g., 3 + ★ = 7)

Use this category if you helped the target student to understand that a symbol such as a box, a shape, or a letter may be used to represent a specific unknown number. **However, if you worked on formulas or equations for modeling a situation, mark that in C2f. Also, please do not record missing addend problems here (e.g., 3 + __ = 7).** Students should be working with a symbol, box, shape, or letter to represent the unknown quantity.

C2f. Understanding and using formulas or equations expressed in symbolic form

Here we mean to include those activities where the target student was given a formula and asked to use it. For example, the student might have been asked to use or explain the terms in the formula for the perimeter of a rectangle, $p = 2(l + w)$. Or the student might have been given the equation $2x + 3 = 7$ and asked to find x using informal or formal methods.

C2g. Expressing a function or sequence as a general rule using words, tables, or formulas

Use this category only when the target student was working on general rules for describing functions or sequences. Functions are sometimes expressed in a table with a place for the general term, for example:

N	1	2	3	4	5	7	12	n
M	2	4	6	8	?	?	?	?

The student might have been asked to find the rule for the function and to use the rule to find additional answers. Here, the student might have used words to describe the rule of the function (e.g., to get the bottom number we need to double the top number) or he or she might have used formulas (e.g., M = 2N). Please use this category to record times when the target student was expressing rules that allowed him to identify any term in a sequence or result of a function – for example, the 100^{th} term in a sequence, or the result for any input to a function**. If the student was only extending a sequence, however, or naming the next or next few terms using an iterative rule (e.g., each term goes up by two), mark that in C2c above.** In this category also include times when the student was asked to write formulas to model a situation such as the following: Isabel gets 2 dollars a week for allowance, plus an extra dollar for each hour of chores she does. The student might express Isabel's allowance as A = 2 + 1C, where A = allowance and C = hours of chores.

Questions A3, B3, and C3:

What did you or the target student use to work on the content that you checked in Question 2?

Please indicate what you or the target student used to work on those specific topics.

a. Numbers or symbols

Mark this category if you or the target student used numbers and/or symbols to work on the focal topic category. For instance, you or the target student might have used the numeral 5 to represent the idea of five objects. Or you or the target student might have worked on addition with fractions using only numbers and symbols. Include in this category worksheets, flashcards, and other purely symbolic means by which students might learn about representations, facts or procedures. **However, if the worksheet or flashcard includes only pictures or diagrams, record this below.**

b. Concrete materials

Mark this category if you or the target student used materials (e.g., pattern blocks, fraction pieces, bean sticks, fingers) to work on the focal topic category. You or the target student might have counted concrete objects, or you or the student might have shown how fractions are equivalent using pattern blocks or diagrams to represent same-sized but differently-named areas.

c. Real-life situations or word problems

Mark this category if you or the target student used contextualized situations, such as needing to find change at the school store, comparing the height of two third-graders, or doubling fractional teaspoons in a recipe. This category includes both situations developed from classroom life **and** word problems found in curriculum materials or written by you or students.

d. Pictures or diagrams

Mark this category if you or the target student used pictures or diagrams to work in the focal topic category. You or the target student might have constructed pictures or diagrams, or they might have been available in curriculum materials or other mathematics materials. If the student worked with number lines or graphs, please record that here. **However, if the diagram was a table or a chart, please mark that below instead.**

e. Tables or charts

Mark this category if you or the target student used tallies, tables, or charts to work in the focal topic category. You or the target student might have constructed tables or charts, or they might have been available in curriculum materials or other mathematics materials.

f. I made explicit links between two or more of these representations

Use this category to describe instruction in which you displayed two or more representations of an idea or procedure and made explicit connections or comparisons between them. For example, in teaching multi-digit subtraction you might have used bundled popsicle sticks to represent the numbers, and in your teaching worked back and forth explicitly among actions on the popsicle sticks, steps of the procedure, and the written notation. Or you might have taught fractions by comparing a regional representation of $\frac{1}{3}$ using pattern blocks to $\frac{1}{3}$ on a number line, asking students how they are the same and different. In addition, the two representations being linked might both be of the same type (e.g., two different concrete representations might be compared). What is important is whether you made explicit connections across the representations. **If you simply showed two representations side by side without comparing or connecting them, please mark both representations above, but do not check this box.**

Questions A4, B4, and C4:

What was the target student asked to do during the work today?

This question asks about the types of tasks in which the target student was engaged today. Although we are asking about the target student here, if you asked the whole class to do something, please consider the target student "engaged" in that task – whether or not you directly heard or saw the target student's thoughts or work, or whether or not the target student appeared to be working. Please only mark a category if students engaged in it for a sustained period of time.

By sustained, we mean more than a few-sentence presentation or a single problem that required only a brief moment to answer. Instead, the students should have had a significant opportunity to engage in the activity described. For instance, during one day students might have done word problems in morning math (4c), listened to your presentation on reducing fractions (4a), practiced reducing fractions (4b), and explained their thinking behind nearly all their answers (4e). Each of these could be marked here. There is no specific time criterion for whether an activity received sustained attention. Please use your judgment, taking into account the time that the student spent on the activity and the importance of the activity to the day's work.

a. Listen to me present the definition for a term or the steps of a procedure

Use this category if students were asked to listen to and understand a term or procedure that you presented. For instance, you might have presented a procedure for multi-digit subtraction with trading and expected students to work on understanding what you did. Or you might have explained what a fraction is and expected students to learn that definition. Presentations may include questions to students; in other words, you may intersperse the presentation of material with questions to students, and still record that work here. **Do not use this category to record student practice with the term or procedure**. Instead, students should have been listening to and making sense of a term or procedure you explained.

b. Perform tasks requiring ideas or methods already introduced to the student

If students were solving problems using procedures shown to them by you today or in prior lessons, or procedures they developed in prior lessons and were today practicing for mastery, please mark this box. Some of the activities included here are student practice for fluency, word problems where the operation was obvious (e.g., 2-digit addition word problems directly following a lesson on 2-digit addition), and practice with terms or definitions (e.g., naming fractions following a lesson on naming fractions, or identifying prime numbers after being shown how). Please use this category to represent work that does not ask students to go beyond what they have seen or done before or been shown in class. **If students were asked to do any new thinking about an idea or develop any approaches on their own, please mark that below under 4d.**

c. Assess a problem and choose a method to use from those already introduced to the student

If you asked students to read or listen to a problem, assess what the problem means, and determine an approach, operation, or method to help solve that problem, record that here. For instance, a student might have read the problem "Michael has 12 apples. Jasmine has 24. How many more apples does Jasmine have?", determined that this problem involves finding a difference or comparing, and decided to use subtraction to model the situation. If the student already knew multi-digit operations procedures and was simply choosing the correct operation, you would record that here. **However, if students are inventing their own methods for solving problems – rather than choosing among previously introduced methods, please mark that below under 4d.**

d. Perform tasks requiring ideas or methods not already introduced to the student

If you asked students to reason about a problem, idea, or method that they had not seen before, please record that here. For instance, you might have asked students who had not yet compared fractions to think and write about whether $\frac{1}{2}$ is larger or smaller than $\frac{1}{3}$. Or you might have asked a student who had not yet done two-digit addition to invent a procedure for doing two-digit addition problems. Or students might have invented an idea, as when a student notices that multiplying a number by $\frac{1}{2}$ is equivalent to dividing a number by 2. What is important here is that students were learning about mathematical topics through their own thinking and reasoning**. If you explained properties of numbers, definitions, or procedures and asked students to reproduce or practice with them, please mark this in one of the categories above.**

e. Explain an answer or a solution method for a particular problem

If you asked students to explain how they solved a problem, please record that here. A student might have, for instance, explained why he chose addition instead of subtraction to model a particular word problem. A student might have explained why a procedure works as it applies within a particular problem, e.g., "23 – 9". I couldn't subtract 9 from the 3, so I needed to trade a ten for ones, renaming the 23 as 10 + 13." A student might have explained an answer based on an estimate or other number-sense reasoning, or the student might have retraced the steps, explaining the procedure she used to solve a particular problem. Students might have done this individually, in a small group, or in pair work**. If students are making explanations beyond a single problem – explaining why a method works generally or explaining what a particular problem shows about a mathematics concept – please mark that in 4f or 4g below.**

f. Analyze similarities and differences among representations, solutions, or methods

If your classroom's work on a problem generated more than one representation, solution or method, <u>and</u> students were asked to analyze and compare similarities and differences among

them explicitly, record that here. Students might have compared, for instance, $\frac{1}{3}$ as represented on a number line and $\frac{1}{3}$ as represented by pattern blocks, asking how these representations are the same or different. Or students might have compared solution methods, observing how one method for solving a two-digit addition problem is different from or similar to another. **Do <u>not</u> mark this category if students were simply asked to approve or correct someone else's work. Please also do not mark this category if students presented two solutions or solution methods and discussed them one by one, but did not explicitly analyze and compare how they were similar to and different from one another.** Students must analyze other students' solution methods and offer reasons related to the underlying mathematical procedures or concepts for this category to apply.

g. Prove that a solution is valid or that a method works for all similar cases

Please record here if you asked students to explain why the approach used to solve a problem made mathematical sense, why the solution was complete, or to generalize beyond particular problems to underlying concepts or principles. This requires that students go beyond explaining why a single answer is correct. For example, if a student developed her own method for reducing fractions, you might have asked "How do you know your method works for any fraction?" You may have asked students to determine that a rule works for every case. Or you might have presented an alternative method of multi-digit multiplication and asked students to determine why it works. This category also includes showing that a solution to a problem is complete – for example, for a problem with multiple solutions, proving that all the solutions have been found, or showing that an answer satisfies all the conditions of a problem. **If you asked students to explain how they got a particular answer, to check their answers, or to go over the steps used in finding the answer to a problem, please record that in 4e above.**

Questions A5, B5, and C5:

Did the target student's work today include any of the following?

This question asks what types of activities the target student was engaged in today. Even though we are asking about the target student here, if you asked the whole class to do something, please consider the target student "engaged" in that task – regardless of whether you heard or saw the target student's thoughts or work, and whether or not the target student appeared to be working. You need not mark an item in this section every day. In fact, you may leave this section blank on days the target student engaged in none of these activities. Please only mark an item if students engaged in it for a sustained period of time.

By sustained, we mean more than a brief paired discussion or a single question that required only a brief moment to answer. Instead, the students should have had a significant opportunity to engage in the activity described. There is no specific time criterion for whether an activity received sustained attention. Please use your judgment, taking into account the time that the student spent on the activity and the importance of the activity to the day's work.

a. Orally answering recall questions

Use this category to describe instruction in which you asked questions during or after your presentation to check for student understanding, or in which you helped the target student review material or practice facts and procedures orally. For instance, some teachers have students answer basic fact questions in "morning math" or "calendar math" and would use this category to represent that work. **Questions here do not require complex thinking**, but are used to provide review and practice, and to check the target student's knowledge.

b. Working on textbook, worksheet, or board work exercises for practice or review

Use this category to describe instruction in which you asked students to review or practice ideas or procedures from prompts in their textbook, worksheets, or from problems written on the board or overhead.

c. Working on problem(s) that have multiple answers or solution methods, or involve multiple steps

Use this category when the target student worked on more complex problems (sometimes referred to as non-routine) that either have more than one clear answer or method or that require more than one step to solve. For example, if an applied problem required students to multiply and then add, mark that here. **If students worked on multi-digit computation, please do not record that here unless the problem had multiple answers, solution methods, or involved multiple steps.**

d. Discussing ideas, problems, solutions, or methods in pairs or small groups

Use this category if you asked the target student to discuss something with a partner or in a small group. The target student should have been exchanging ideas about some specific

problem, solution, or method. **If your students are routinely seated in pairs or small groups, please only mark this category when you specifically asked them to discuss an idea, procedure, solution or method together.**

e. Using flashcards, games, or computer activities to improve recall or skill

Use this category if you asked the target student to use flashcards, games, or computer activities to improve their recall of facts or definitions, or increase their skill with computation. For instance, you might have played "Around the World" to practice multiplication facts, or had students work in pairs to quiz one another using flashcards.

f. Writing extended explanations of mathematical ideas, solutions, or methods

Use this category if you asked the target student to write in an extended way about a mathematics topic. For example, you might have asked the target student to write about how he or she solved a problem or why he or she used a specific approach. What is considered "extended" varies by the grade level of the student. **However, please do <u>not</u> include times when the target student was merely writing an answer as a complete sentence, or copying a definition or explanation that you provided.**

g. Working on an investigation, problem, or project over an extended period of time

Use this category when the target student worked on an extended mathematics activity that lasted two or more days. Here we include activities that are designed to be more involved and last more than a single mathematics session. **However, if the target student is simply finishing a worksheet or an activity that he or she began the day before, please do <u>not</u> mark this category.**

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