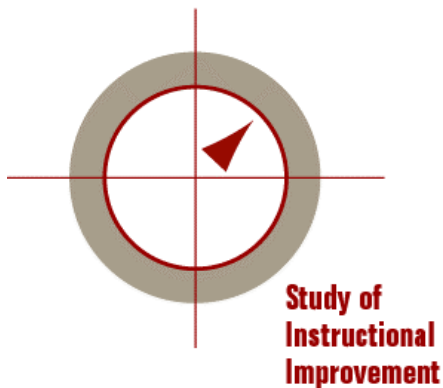


STUDY OF INSTRUCTIONAL IMPROVEMENT

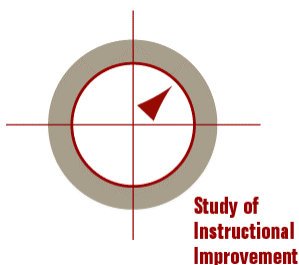
MEASURING TEACHERS' CONTENT KNOWLEDGE FOR TEACHING

ELEMENTARY MATHEMATICS RELEASE ITEMS 2002



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Dear Colleague:

Thank you for your interest in our survey items measuring teachers' knowledge for teaching mathematics. To orient you to the items and their potential use, we explain their development, intent, and design in this letter.

The effort to design survey items measuring teachers' knowledge for teaching mathematics grew out of the unique needs of the *Study of Instructional Improvement (SII)*. SII is investigating the design and enactment of three leading whole school reforms and these reforms' effects on students' academic and social performance. As part of this research, lead investigators realized a need not only for measures which represent school and classroom processes (e.g., school norms, resources, teachers' instructional methods) but also teachers' facility in using disciplinary knowledge in the context of classroom teaching. Having such measures will allow SII to investigate the effects of teachers' knowledge on student achievement, and track changes in teachers' knowledge as a result of engagement with whole school and other reforms. While many potential methods for exploring and measuring teachers' content knowledge exist (i.e., interviews, observations, structured tasks), we elected to focus our efforts on developing survey measures because of the large number of teachers (nearly 2000) participating in SII.

Beginning in 1999, we undertook the development of such survey measures. Using theory, research, the study of curriculum materials and student work, and our experience, we wrote items we believe represent some of the competencies teachers use in teaching elementary mathematics – representing numbers, interpreting unusual student answers or algorithms, anticipating student difficulties with material. With the assistance of the University of California Office of the President¹, we piloted these items as part of an on-going evaluation of the state's Mathematics Professional Development Institutes. This California collaboration developed into a sister project to SII, *Learning Mathematics for Teaching*.

¹ Elizabeth Stage, Patrick Callahan, Rena Dorph, principals.

We are publicly releasing a small set of items from these projects' initial efforts to write and pilot survey measures. We believe these items can be useful in many different contexts: as open-ended prompts which allow for the exploration of teachers' reasoning about mathematics and student thinking; as materials for professional development or teacher education; as exemplars of the kinds of mathematics teachers must know to teach. We encourage their use in such contexts. However, this particular set of items is, as a group, NOT appropriate for use as an overall measure, or scale, representing teacher knowledge. The set of items is instead merely representative of the kinds of items which may eventually be used in creating scales with adequate reliabilities which can be used in statistical work.

We ask users to keep in mind that these items represent steps in the processes of developing measures. None are perfect, although some are better than others. If you have comments or ideas about these items, please feel free to contact one of us by email at the addresses below.

These items are the result of years of thought and development, including both qualitative investigations of the content teachers use to teach elementary mathematics, and quantitative field trials with large numbers of survey items and participating teachers. Because of the intellectual effort put into these items by SII investigators, we ask that *all* users of these items satisfy the following requirements:

1) Please request permission from SII for any use of these items. To do so, contact Heather Hill at hhill@umich.edu. Include a brief description of how you plan to use the items, and if applicable, what written products might result.

2) In any publications, grant proposals, or other written work which results from use of these items, please cite the development efforts which took place at SII by referencing this document:

Ball, D. L., Hill, H.C., Rowan, B., Schilling, S. (2002) Measuring teachers' content knowledge for teaching: Elementary mathematics release items, 2002. Ann Arbor, Michigan: Study of Instructional Improvement.

You can also check the SII website (<http://www.sii.soe.umich.edu/>) for more information about this effort.

Again, thank you for your interest in these items.

Sincerely,

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**Study of Instructional Improvement
Measuring Teachers' Content Knowledge for Teaching
Elementary Mathematics Release Items
2002
CONTENT KNOWLEDGE ITEMS²**

1. Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

Which statement(s) should the sisters select as being true? (Mark YES, NO, or I'M NOT SURE for each item below.)

	Yes	No	I'm not sure
a) 0 is an even number.	1	2	3
b) 0 is not really a number. It is a placeholder in writing big numbers.	1	2	3
c) The number 8 can be written as 008.	1	2	3

² **All measures copyright 2002, Study of Instructional Improvement (SII)/Consortium for Policy Research in Education (CPRE). Not for reproduction or use without written consent of SII. Measures development supported by NSF grant REC-9979873, and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.**

2. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
$\begin{array}{r} 35 \\ \times 25 \\ \hline 125 \\ +75 \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 175 \\ +700 \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 25 \\ 150 \\ 100 \\ +600 \\ \hline 875 \end{array}$

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

3. Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)

- a) Four is an even number, and odd numbers are not divisible by even numbers.**
- b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).**
- c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.**
- d) It only works when the sum of the last two digits is an even number.**

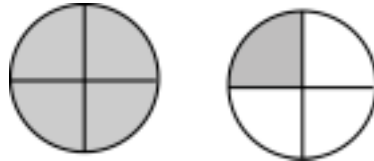
4. Ms. Chambreaux's students are working on the following problem:

Is 371 a prime number?

As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

- a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.**
- b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.**
- c) Check to see whether 371 is divisible by any prime number less than 20.**
- d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.**

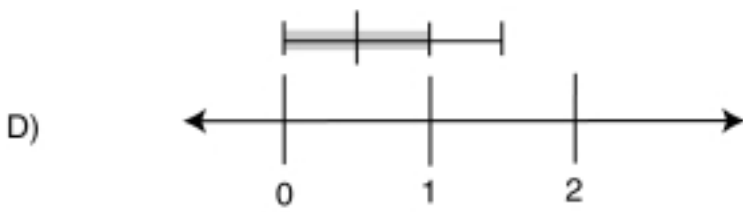
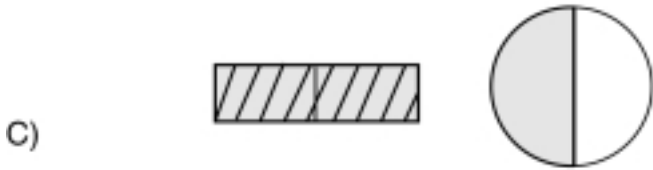
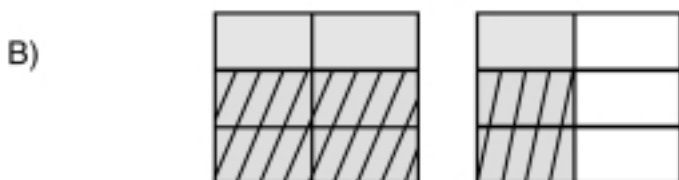
5. Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)



- a) $5/4$**
- b) $5/3$**
- c) $5/8$**
- d) $1/4$**

6. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do not represent multiplication of fractions appropriately.

Which model below cannot be used to show that $1\frac{1}{2} \times \frac{2}{3} = 1$? (Mark ONE answer.)



7. Which of the following story problems could be used to illustrate

$1\frac{1}{4}$ divided by $\frac{1}{2}$? (Mark YES, NO, or I'M NOT SURE for each possibility.)

	Yes	No	I'm not sure
a) You want to split $1\frac{1}{4}$ pies evenly between two families. How much should each family get?	1	2	3
b) You have \$1.25 and may soon double your money. How much money would you end up with?	1	2	3
c) You are making some homemade taffy and the recipe calls for $1\frac{1}{4}$ cups of butter. How many sticks of butter (each stick = $\frac{1}{2}$ cup) will you need?	1	2	3

8. As Mr. Callahan was reviewing his students' work from the day's lesson on multiplication, he noticed that Todd had invented an algorithm that was different from the one taught in class. Todd's work looked like this:

$$\begin{array}{r} 983 \\ \times 6 \\ \hline 488 \\ + 5410 \\ \hline 5898 \end{array}$$

What is Todd doing here? (Mark ONE answer.)

- a) Todd is regrouping ("carrying") tens and ones, but his work does not record the regrouping.**
- b) Todd is using the traditional multiplication algorithm but working from left to right.**
- c) Todd has developed a method for keeping track of place value in the answer that is different from the conventional algorithm.**
- d) Todd is not doing anything systematic. He just got lucky – what he has done here will not work in most cases.**

STUDENT THINKING ITEMS

9. Mr. Garrett's students were working on strategies for finding the answers to multiplication problems. Which of the following strategies would you expect to see some elementary school students using to find the answer to 8×8 ? (Mark YES, NO, or I'M NOT SURE for each strategy.)

	Yes	No	I'm not sure
a) They might multiply $8 \times 4 = 32$ and then double that by doing $32 \times 2 = 64$.	1	2	3
b) They might multiply $10 \times 10 = 100$ and then subtract 36 to get 64.	1	2	3
c) They might multiply $8 \times 10 = 80$ and then subtract 8×2 from 80: $80 - 16 = 64$.	1	2	3
d) They might multiply $8 \times 5 = 40$ and then count up by 8's: 48, 56, 64.	1	2	3

10. Students in Mr. Hayes' class have been working on putting decimals in order. Three students — Andy, Clara, and Keisha — presented 1.1, 12, 48, 102, 31.3, .676 as decimals ordered from least to greatest. What error are these students making? (Mark ONE answer.)

- a) They are ignoring place value.**
- b) They are ignoring the decimal point.**
- c) They are guessing.**
- d) They have forgotten their numbers between 0 and 1.**
- e) They are making all of the above errors.**

11. You are working individually with Bonny, and you ask her to count out 23 checkers, which she does successfully. You then ask her to show you how many checkers are represented by the 3 in 23, and she counts out 3 checkers. Then you ask her to show you how many checkers are represented by the 2 in 23, and she counts out 2 checkers. What problem is Bonny having here? (Mark ONE answer.)

- a) Bonny doesn't know how large 23 is.**
- b) Bonny thinks that 2 and 20 are the same.**
- c) Bonny doesn't understand the meaning of the places in the numeral 23.**
- d) All of the above.**

12. Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students focused on particular difficulties that they are having with adding columns of numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

I)	$\begin{array}{r} 1 \\ 38 \\ 49 \\ + 65 \\ \hline 142 \end{array}$	II)	$\begin{array}{r} 1 \\ 45 \\ 37 \\ + 29 \\ \hline 101 \end{array}$	III)	$\begin{array}{r} 1 \\ 32 \\ 14 \\ + 19 \\ \hline 64 \end{array}$
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Which have the same kind of error? (Mark ONE answer.)

- a) I and II**
- b) I and III**
- c) II and III**
- d) I, II, and III**

13. Ms. Walker's class was working on finding patterns on the 100's chart. A student, LaShantee, noticed an interesting pattern. She said that if you draw a plus sign like the one shown below, the sum of the numbers in the vertical line of the plus sign equals the sum of the numbers in the horizontal line of the plus sign (i.e., $22 + 32 + 42 = 31 + 32 + 33$). Which of the following student explanations shows sufficient understanding of why this is true for all similar plus signs? (Mark YES, NO or I'M NOT SURE for each one.)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

	Yes	No	I'm not sure
a) The average of the three vertical numbers equals the average of the three horizontal numbers.	1	2	3
b) Both pieces of the plus sign add up to 96.	1	2	3
c) No matter where the plus sign is, both pieces of the plus sign add up to three times the middle number.	1	2	3
d) The vertical numbers are 10 less and 10 more than the middle number.	1	2	3

14. Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students around particular difficulties that they are having with subtracting from large whole numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

I	II	III
$\begin{array}{r} 4\ 12 \\ \cancel{502} \\ - 6 \\ \hline 406 \end{array}$	$\begin{array}{r} 4\ 15 \\ \cancel{38008} \\ - 6 \\ \hline 34009 \end{array}$	$\begin{array}{r} 6\ 98\ 15 \\ \cancel{7808} \\ - 7 \\ \hline 6988 \end{array}$

Which have the same kind of error? (Mark ONE answer.)

- a) I and II
- b) I and III
- c) II and III
- d) I, II, and III

15. Takeem's teacher asks him to make a drawing to compare $\frac{3}{4}$ and $\frac{5}{6}$. He draws the following:



and claims that $\frac{3}{4}$ and $\frac{5}{6}$ are the same amount. What is the most likely explanation for Takeem's answer? (Mark ONE answer.)

- a) Takeem is noticing that each figure leaves one square unshaded.
- b) Takeem has not yet learned the procedure for finding common denominators.
- c) Takeem is adding 2 to both the numerator and denominator of $\frac{3}{4}$, and he sees that that equals $\frac{5}{6}$.
- d) All of the above are equally likely.

16. A number is called “abundant” if the sum of its proper factors exceeds the number. For example, 12 is abundant because $1 + 2 + 3 + 4 + 6 > 12$. On a homework assignment, a student incorrectly recorded that the numbers 9 and 25 were abundant. What are the most likely reason(s) for this student’s confusion? (Mark YES, NO or I’M NOT SURE for each.)

	Yes	No	I’m not sure
a) The student may be adding incorrectly.	1	2	3
b) The student may be reversing the definition, thinking that a number is “abundant” if the number exceeds the sum of its proper factors.	1	2	3
c) The student may be including the number itself in the list of factors, confusing proper factors with factors.	1	2	3
d) The student may think that “abundant” is another name for square numbers.	1	2	3